Math Contest: Probability

Solve each problem. Show your work and circle your final answer.

1. Two fair 4-sided dice, each numbered 1 through 4, are rolled. What is the probability that the sum of the two numbers is a multiple of 3? Express your answer as a common fraction.

2. An integer is chosen at random from 1 to 100, inclusive. What is the probability that it is neither a perfect square nor a perfect cube? Express your answer as a common fraction.

3. A 3-digit code is formed by choosing each digit at random from 0 through 9 (digits may repeat). What is the probability that all three digits are different? Express your answer as a common fraction.

4. Two teams of equal ability play a best-of-5 series. The first team to win 3 games wins the series. What is the probability that the series ends after exactly 4 games? Express your answer as a common fraction.

5. A positive integer less than 100 is chosen at random. What is the probability that it is a perfect cube? Express your answer as a common fraction.

6. Two fair 6-sided dice are rolled. What is the probability that the sum is 7 or 11? Express your answer as a common fraction.

7. An integer is chosen at random from 1 to 100, inclusive. What is the probability that it is a multiple of 7 but not a multiple of 2? Express your answer as a common fraction.

8. Three fair 6-sided dice are rolled. What is the probability that at least one of them shows a 6? Express your answer as a common fraction.

9. Three boys and five girls line up in a random order. What is the probability that the three boys are standing next to each other? Express your answer as a common fraction.

10. Three marbles are drawn at random, without replacement, from a bag with 4 red marbles and 4 blue marbles. What is the probability that all three marbles are the same color? Express your answer as a common fraction.

(11) A club has 5 boys and 4 girls. Three members are chosen at random for a committee. What is the probability that the committee has exactly two boys? Express your answer as a common fraction.

(12) A fair coin is flipped until it lands heads. What is the probability that it takes exactly 3 flips? Express your answer as a common fraction.

(13) A fair 6-sided die is rolled. If the result is 1 or 2, you win 9 dollars. If the result is 3 or 4, you win 3 dollars. If the result is 5 or 6, you lose 6 dollars. What is your expected win, in dollars?

(14) An integer is chosen at random from 1 to 20, inclusive. What is the probability that it is prime? Express your answer as a common fraction.

(15) A positive divisor of 100 is chosen at random. What is the probability that it is a multiple of 5? Express your answer as a common fraction.

(16) A fair 6-sided die is rolled twice. What is the probability that the second number rolled is greater than the first? Express your answer as a common fraction.

(17) Three fair coins are flipped and one fair 6-sided die is rolled. What is the probability that all three coins show heads and the die shows a 6? Express your answer as a common fraction.

(18) A bag contains 5 red marbles, 3 blue marbles and 2 green marbles. Two marbles are drawn at random without replacement. What is the probability that both are red? Express your answer as a common fraction.

(19) What is the expected value of the number rolled on a fair 6-sided die? Express your answer as a decimal.

(20) One letter of the alphabet is chosen at random. What is the probability that it is one of the letters in the word PROBABILITY? Express your answer as a common fraction.